GRAVITATONAL LENSING AND EINSTEIN RINGS

Do Thi Hoai

HCM city, November 21st , 2011

Content

1. Introduction
2. Theoretical basis
3. Bending of light
4. Einstein rings
5. Conclusions

Introduction

A direct consequence of general relativity: light bends in the vicinity of a gravity field. Light emitted by a distant object and travelling near a very massive object in the foreground will appear to come from a point away from the real source and produce effects of mirage and of light concentration generally referred to as **gravitational lensing**.

One commonly distinguishes between three types of gravitational lensing: strong, weak and micro.

Strong lensing: when there are easily visible distortions such as the formation of Einstein rings, arcs, and multiple images.

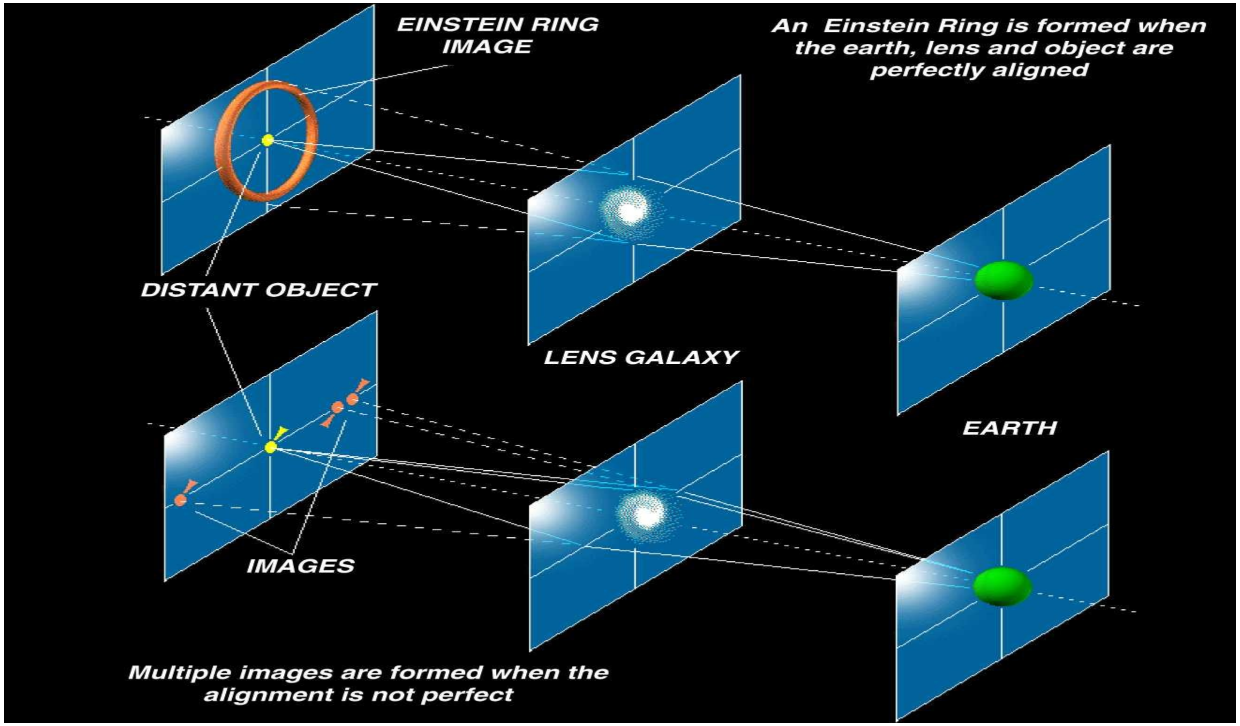
Weak lensing: when the distortions of the background sources are too small, say only a few percent, to allow for an analysis in terms of single source-lens pairs but sufficiently numerous to allow for a statistical analysis.

Microlensing refers to cases where the effect is too small to produce visible distortions in shape, but the amount of light received from a background source is observed to change with time while the lens passes in front of the source.

Strong lensing

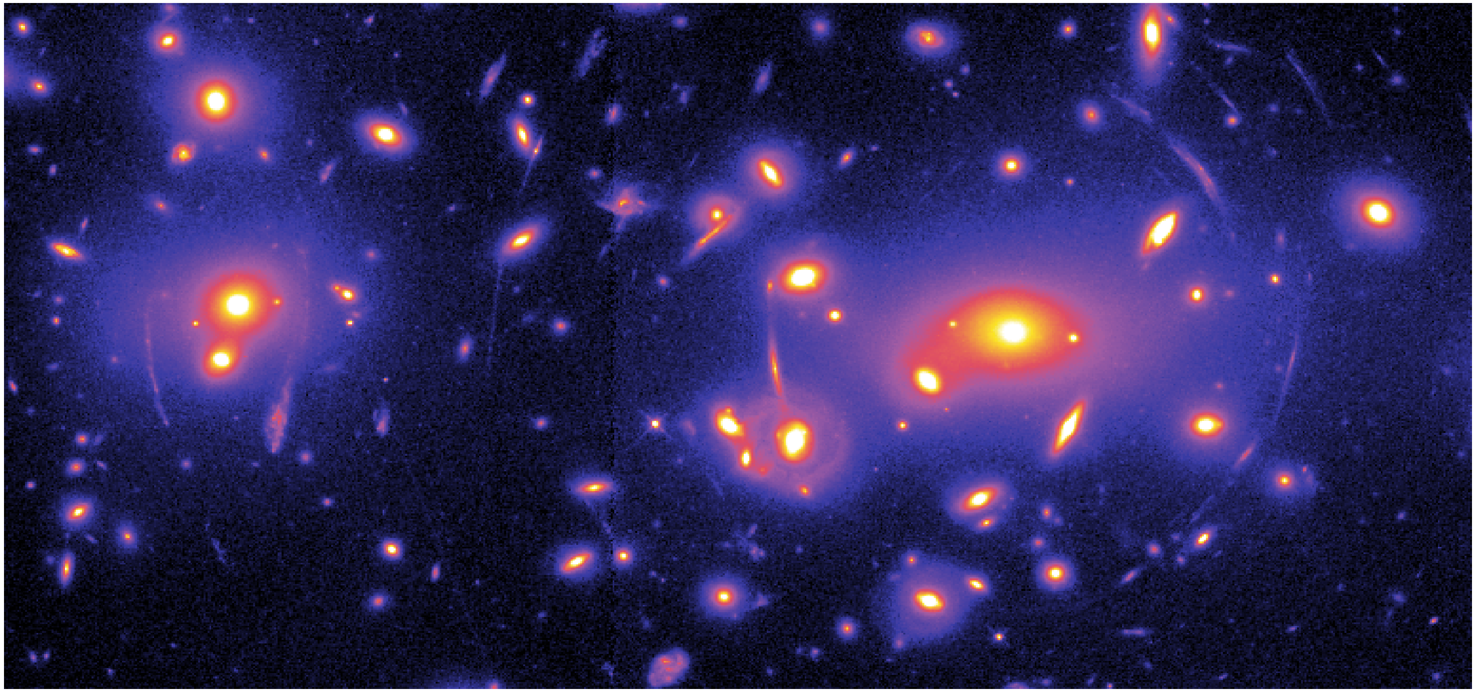
Ideally, an Einstein ring occurs when the lens and the source are both spherical and exactly on the line of sight of the observer.

When the lens or the source are not spherical or when the alignment is not perfect, one observes multiple images of the same source or partial arcs scattered around the lens. The number and shape of these depend upon the relative positions of the source, lens, and observer, and the shape of the gravitational well of the lensing object.



Weak lensing

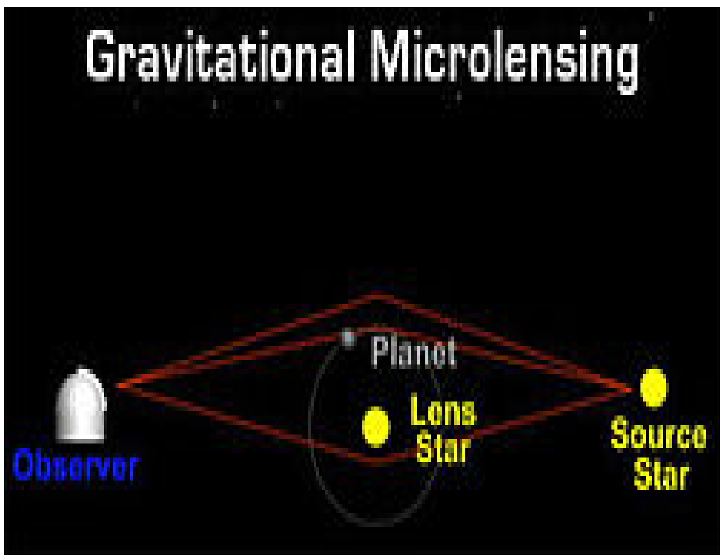
Observed: preferred stretching of the background objects perpendicular to the direction to the center of the lens. By measuring the shapes and orientations of large numbers of distant galaxies, their orientations can be averaged to measure the shear of the lensing field in any region. This, in turn, can be used to reconstruct the mass distribution in the area: in particular, the distribution of dark matter can be reconstructed.



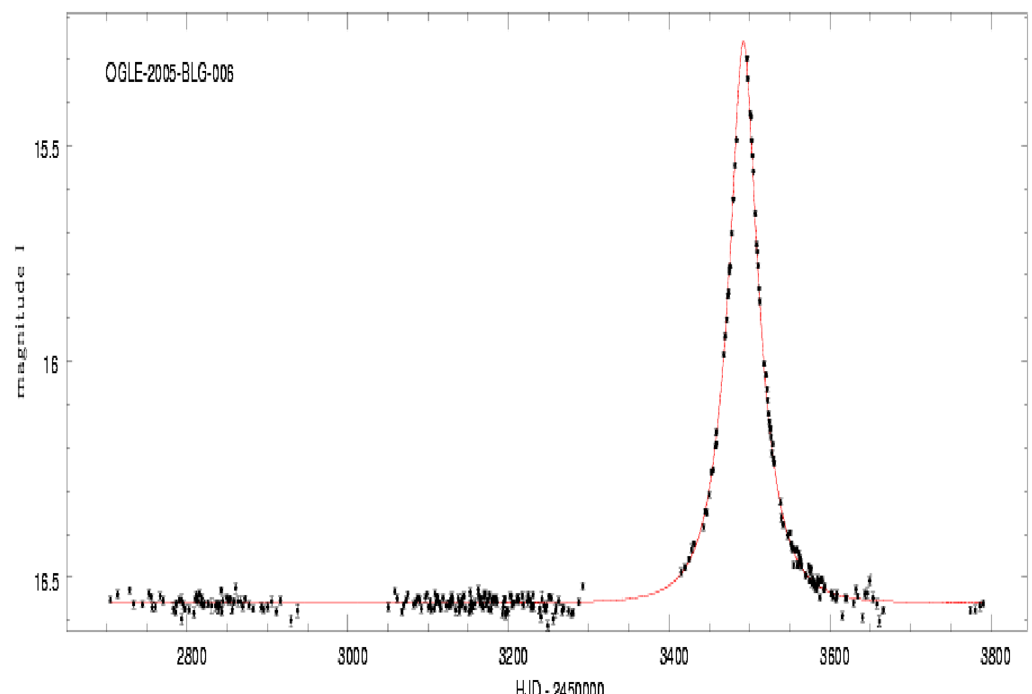
Distant galaxies lensed by Cluster Abell 2218

Microlensing

Refers to cases where the effect is too small to produce visible distortions in shape, but the amount of light received from a background source is observed to change with time while the lens passes in front of the source (in the cases of strong and weak lensing, both source and lens are fixed). Microlensing has been used to search for brown dwarfs in order to evaluate their contribution to dark matter and, more recently, to search for exoplanets with much success.



Detection of exoplanets by gravitational microlensing



A typical microlensing event (OGLE, 2005)

Special relativity: Lorentz transformations

The basis of special relativity is the so-called relativity principle according to which the laws of nature are the same in two frames in uniform movement with respect to each other, usually referred to as inertial frames.

The Lorentz transformation:

$$x' = x \cosh\alpha + t \sinh\alpha$$
$$t' = x \sinh\alpha + t \cosh\alpha$$

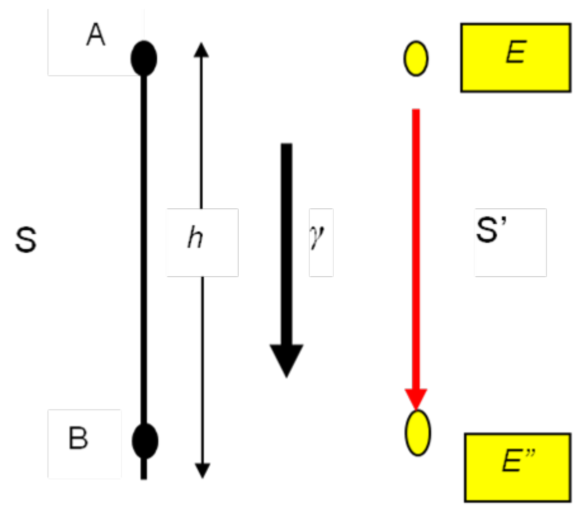
velocity $\beta = \tanh\alpha$

Energy E and momentum p form a four vector:

$$E^2 - p^2 = m^2$$

(m being the rest mass of the particle, a scalar)

Gravity of photons



General relativity extends the relativity principle from inertial frames to frames in free fall

Consider a homogeneous gravity field. Send a photon of energy E from A to B.

A photon being massless $\rightarrow$ $E=p$.

At B, the Lorentz transformation reads $E' = E\cosh\alpha + p\sinh\alpha$

where $\tanh\alpha = \gamma h$ $\rightarrow$ $E'=E+E\gamma h$

Accordingly, when a star having a mass M and a radius R emits a photon of frequency ν, this photon is red shifted when it reaches far distances by an amount (remember that $E=\hbar\nu$) $\Delta\nu/\nu=\Delta E/E=GM/R$.

Schwarzschild metric

The Schwarzschild metric applies in empty space around a massive body.

Schwarzschild metric has the form:

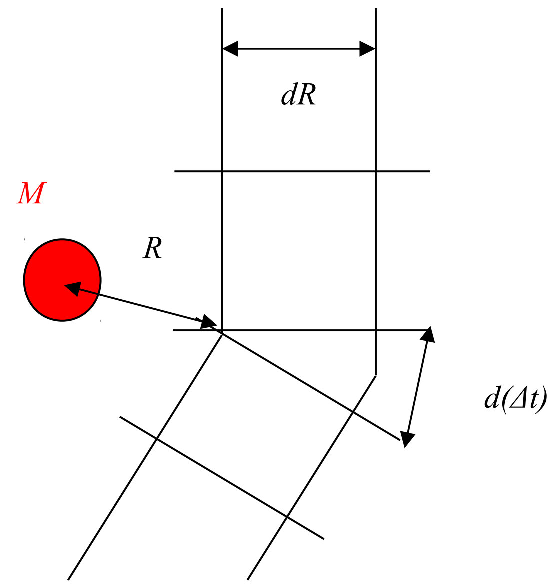
$$ds^2 = (1 - 2MG/r)dt^2 - (1 - 2MG/r)^{-1} dr^2 - r^2(\sin^2\theta d\varphi^2 + d\theta^2)$$

A singularity occurs at $R_{Schwarzschild} = 2MG$ where the escape velocity is equal to the light velocity (equivalently, where a body falling from infinity, originally with zero velocity, has been accelerated to the light velocity). It corresponds to black holes.

Bending of light

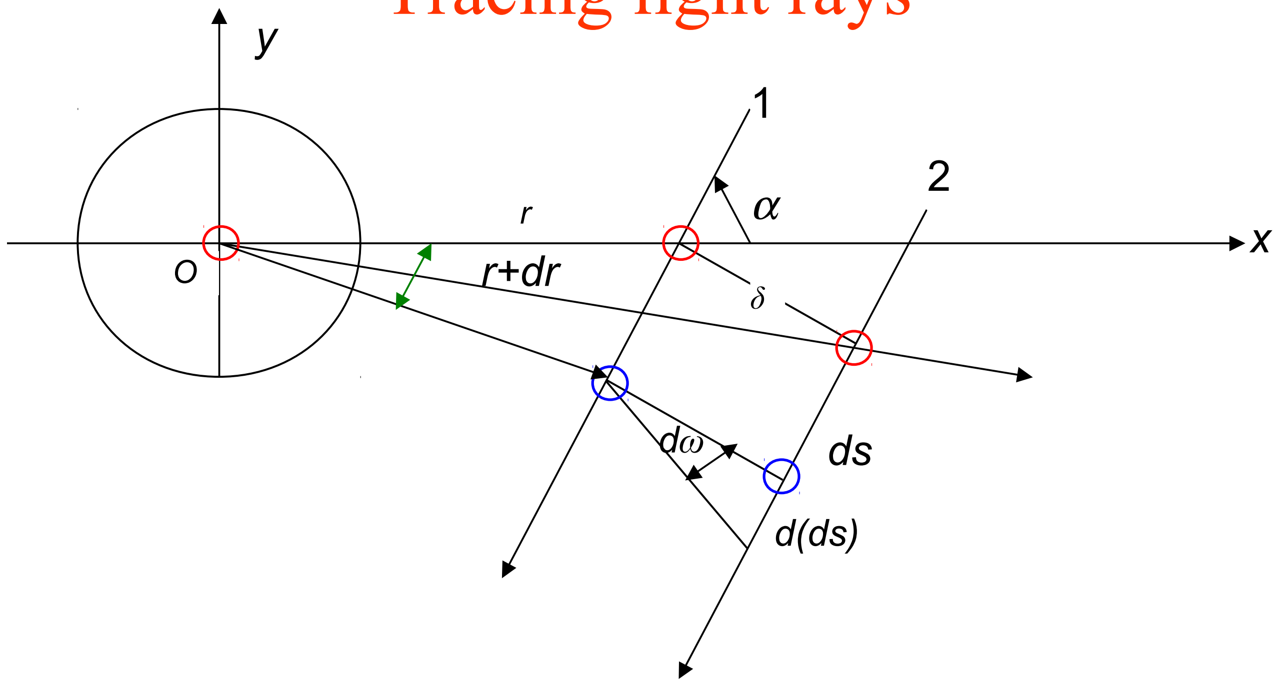
Light travels more slowly in a gravity field, the delay on Earth of light emitted by the Sun amounts to *50 μs* (10^{-7}). This has been checked by sending a radar signal from the Earth to Venus and back. This effect is called gravitational delay.

A consequence of gravitational delay is the bending of light in the vicinity of a massive body, light travelling more slowly closer to the body than farther away. The bending angle in the approximation of small bending is $4MG/R$, M being the mass of the massive body and R being the closest distance of approach of the light ray.



It amounts to nearly 2 seconds of arc in the case of the Sun, which may be thought of as being a weak lens with focal length equal to its radius divided by this angle of deflection, namely some 550 AU.

Tracing light rays



We start with a photon at a distance r from the centre O of a spherically symmetric lens of mass M and radius R.

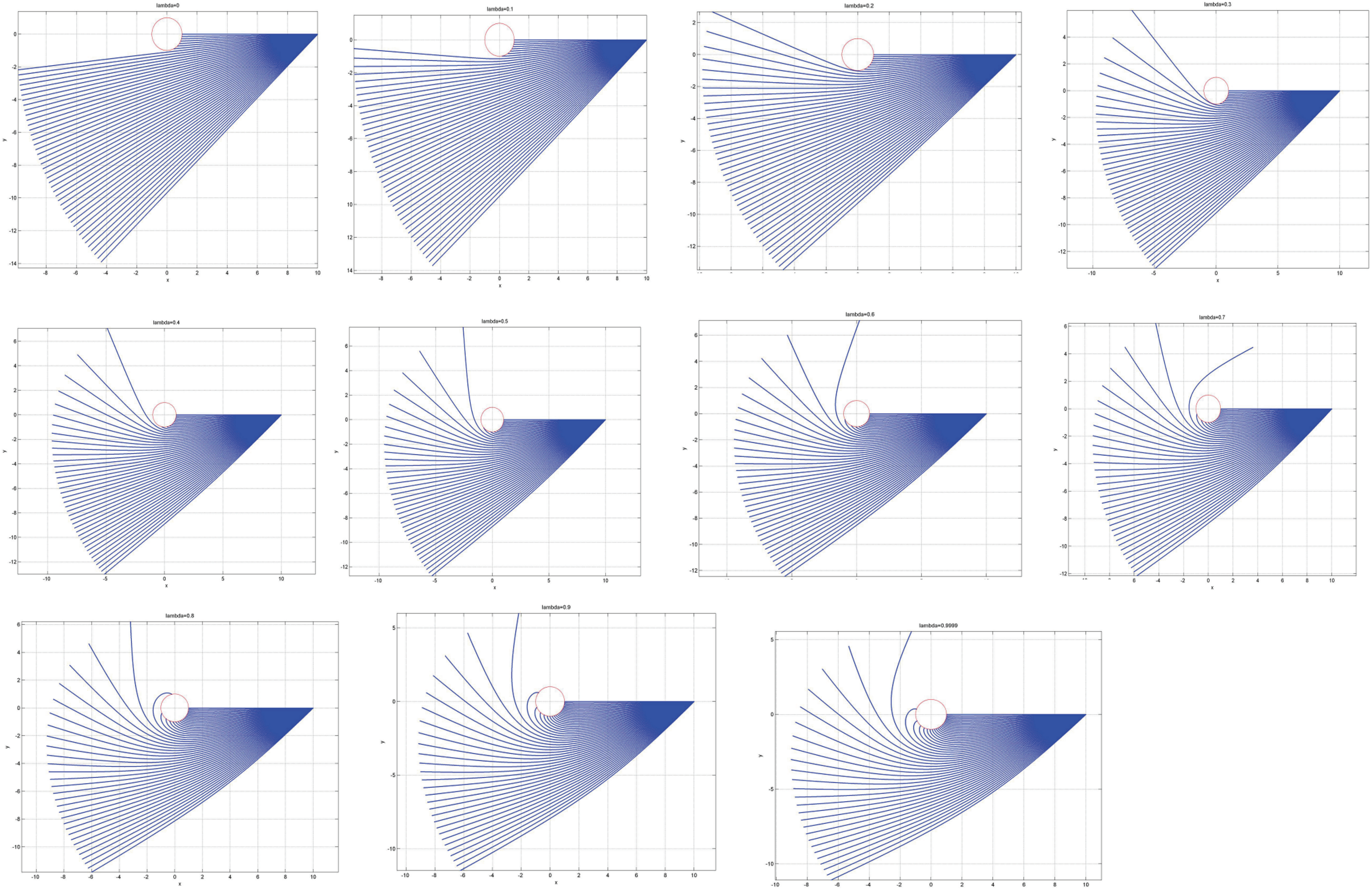
The Schwarzschild radius of the lens is $R^*=2GM$.

We use R as unit length and define $R^*=\lambda R$, $r=\rho R$ and $d\sigma=ds/R$

An important parameter is $\zeta= \lambda/\rho=R^*/r$

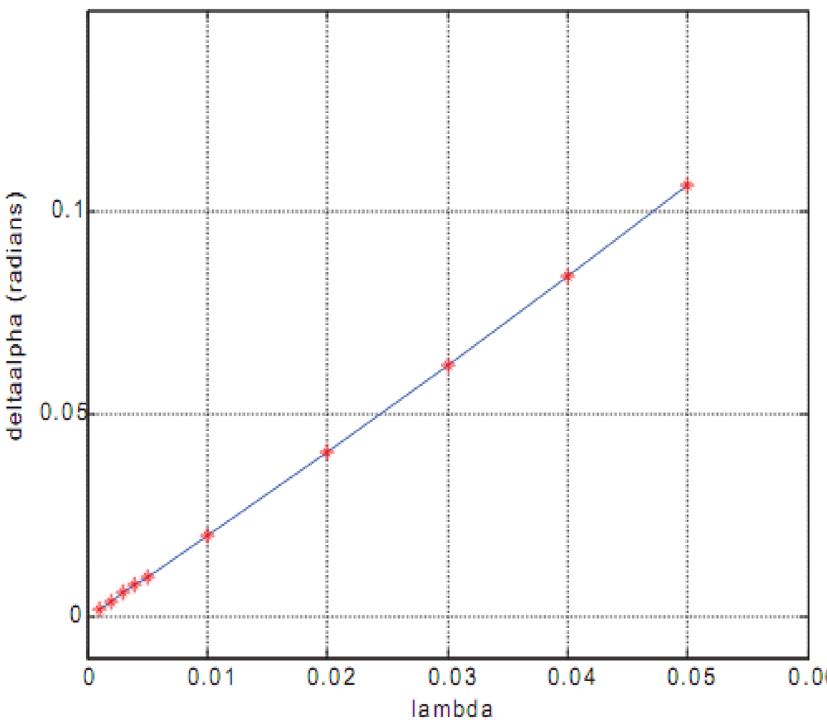
Light rays are traced in steps of $d\sigma = 0.01$ using the relation:

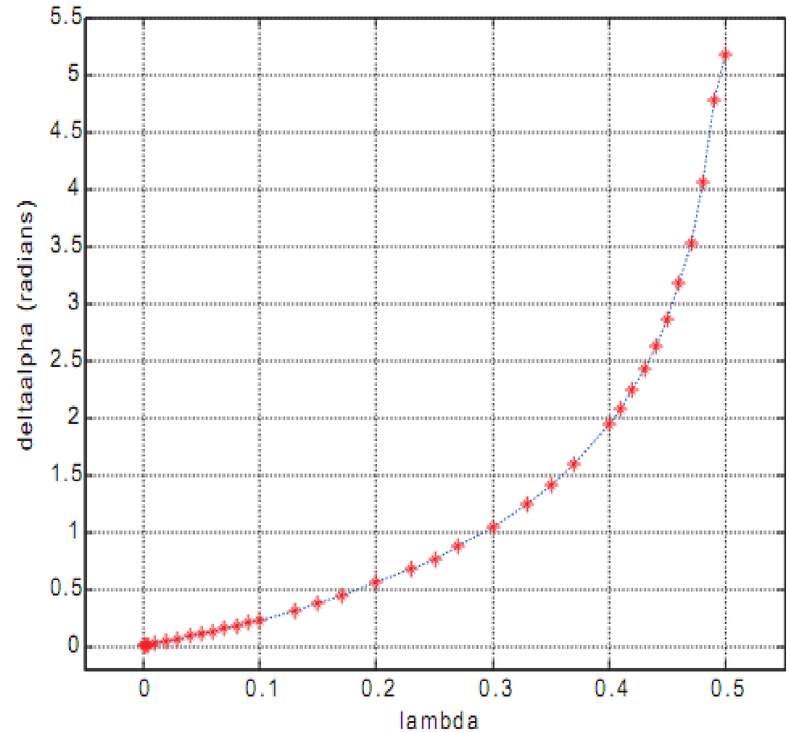
$$d\alpha/d\sigma=\sin\alpha(\rho-2\lambda)/(\rho[\rho-\lambda])=(\sin\alpha/\rho)([1-2\zeta]/[1-\zeta])$$



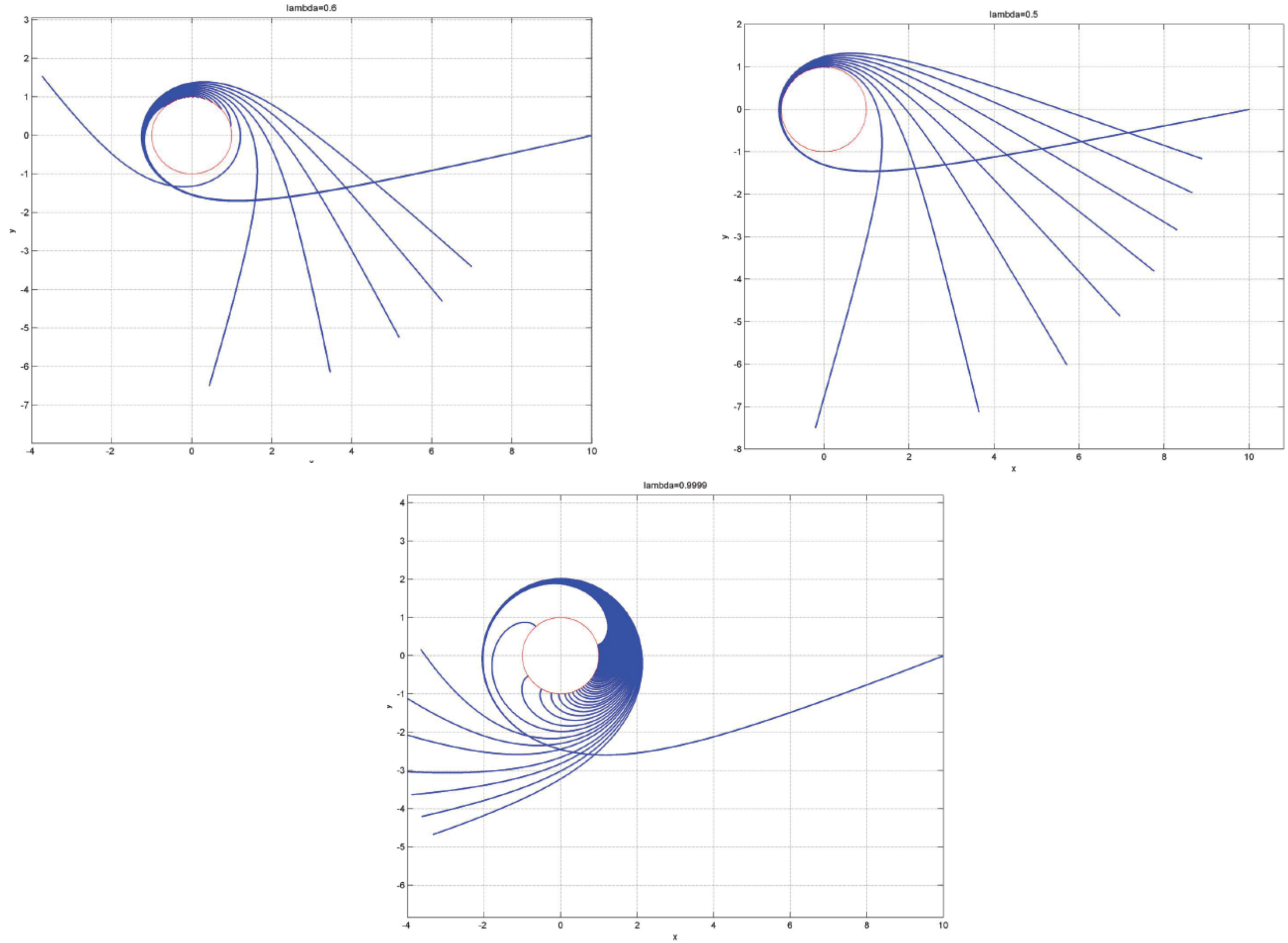
Light rays traced for λ=0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 0.9999

We expect rays in the vicinity of $\alpha_0 = \alpha_{limit}$ to be bent by $4MG/R=2\lambda$ in the small bending approximation. This is indeed what we find with the simulation but as soon as λ exceeds a few percent, bending increases much faster than linearly and the light ray may indeed curl around the source when approaching the black hole limit.





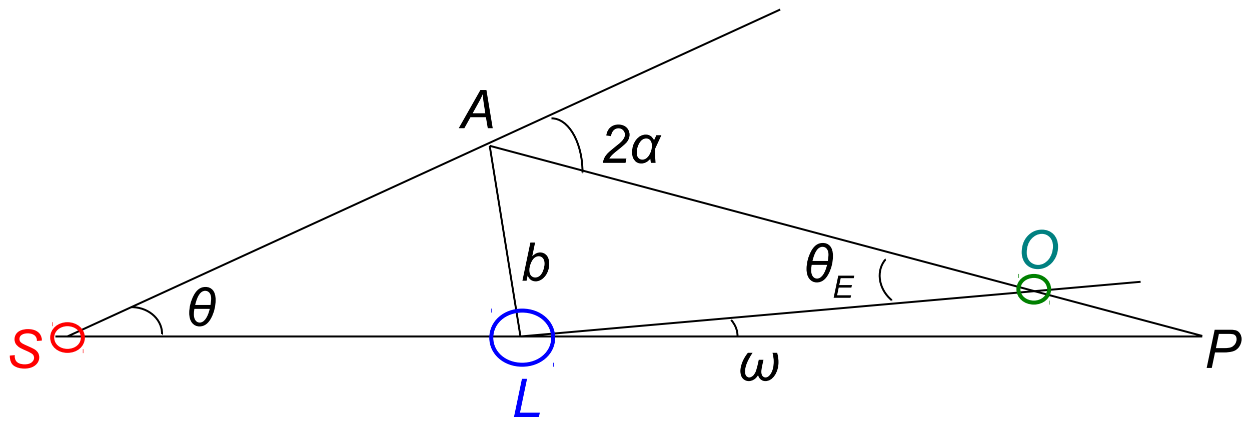
Dependence of $\Delta\alpha$ on λ for two different ranges of λ.



Three examples of rays in the vicinity of $\alpha_0 = \alpha_{limit}$ *.*

The values of λ *are 0.5, 0.6 and 1 respectively.*

Einstein rings



We consider the lensing of a remote quasar S by a foreground spherical galaxy L.

Assuming: Mass of the lens: $M=10^{12}M_{☼}$ => $R_{Schwarzschild}$ ~$3\ 10^{12}km=10^{7}\ ls=3\ 10^{-1}\ ly$.

Radius of the lens: $R = 3\ 10^{3}\ ly$, meaning $\lambda=10^{-4}$,

$LS=10^{9}\ ly$ and $OL=10^{8}\ ly$.

The detector resolution is $\zeta=0.2\ ppm$.

The angle ω (in ppm) measures the misalignment of the observer with respect to the source-lens line.

Defining: $\sigma_S=R/SL$, $\sigma_O=R/OL$, $\sigma_{OS}=\sigma_S+\sigma_O=OS/(SL\times OL)$, $k=(1+\sigma_O/\sigma_S)^{-1}=\sigma_S/\sigma_{OS}$

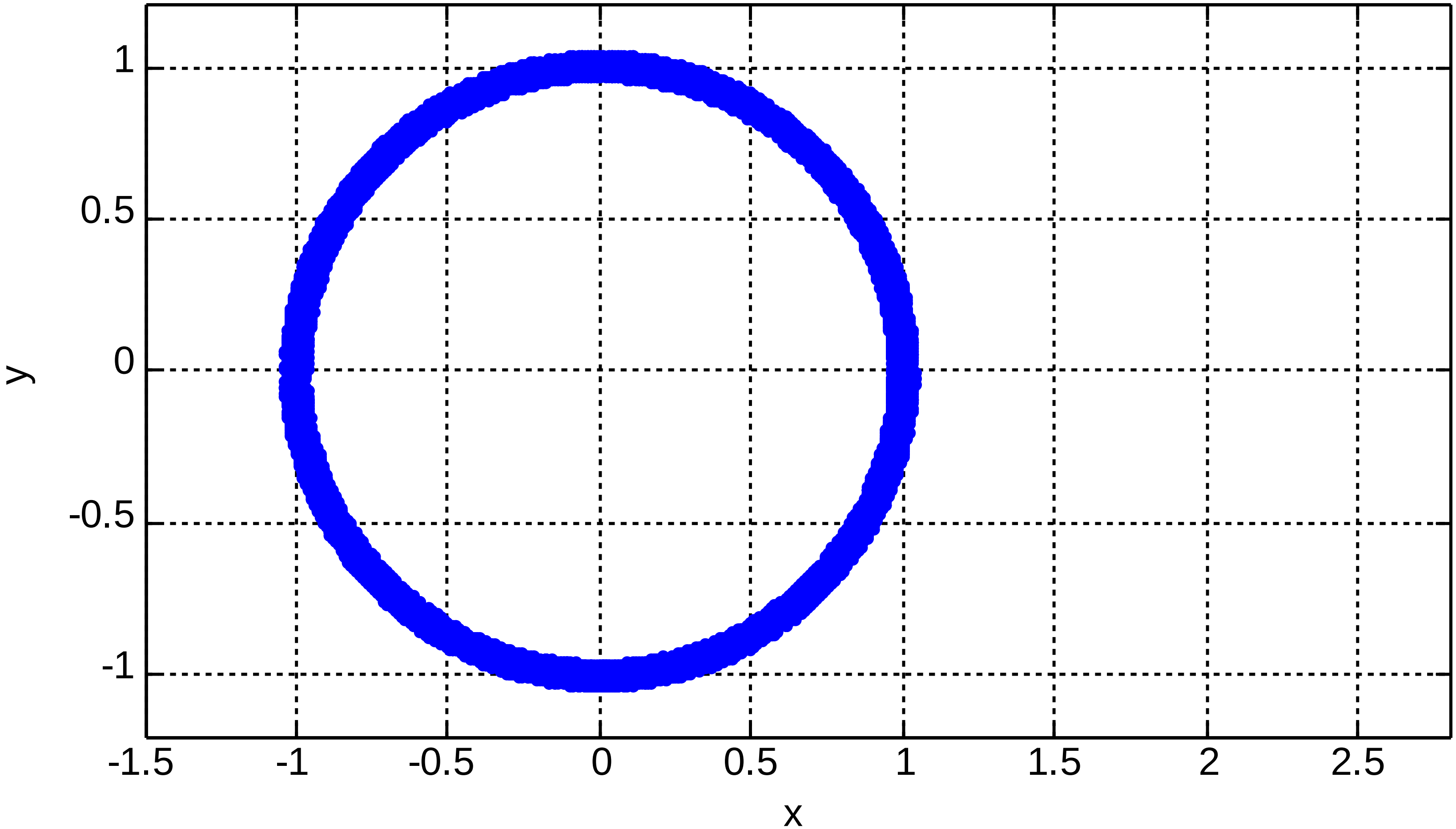
In the SOL plane, two rays reach from S to O, on either sides of the lens ($\theta_{\pm}$).

$\theta_{\pm}=½\{k\omega\pm\sqrt{k^2\omega^2+8k\lambda\sigma_S}\}$

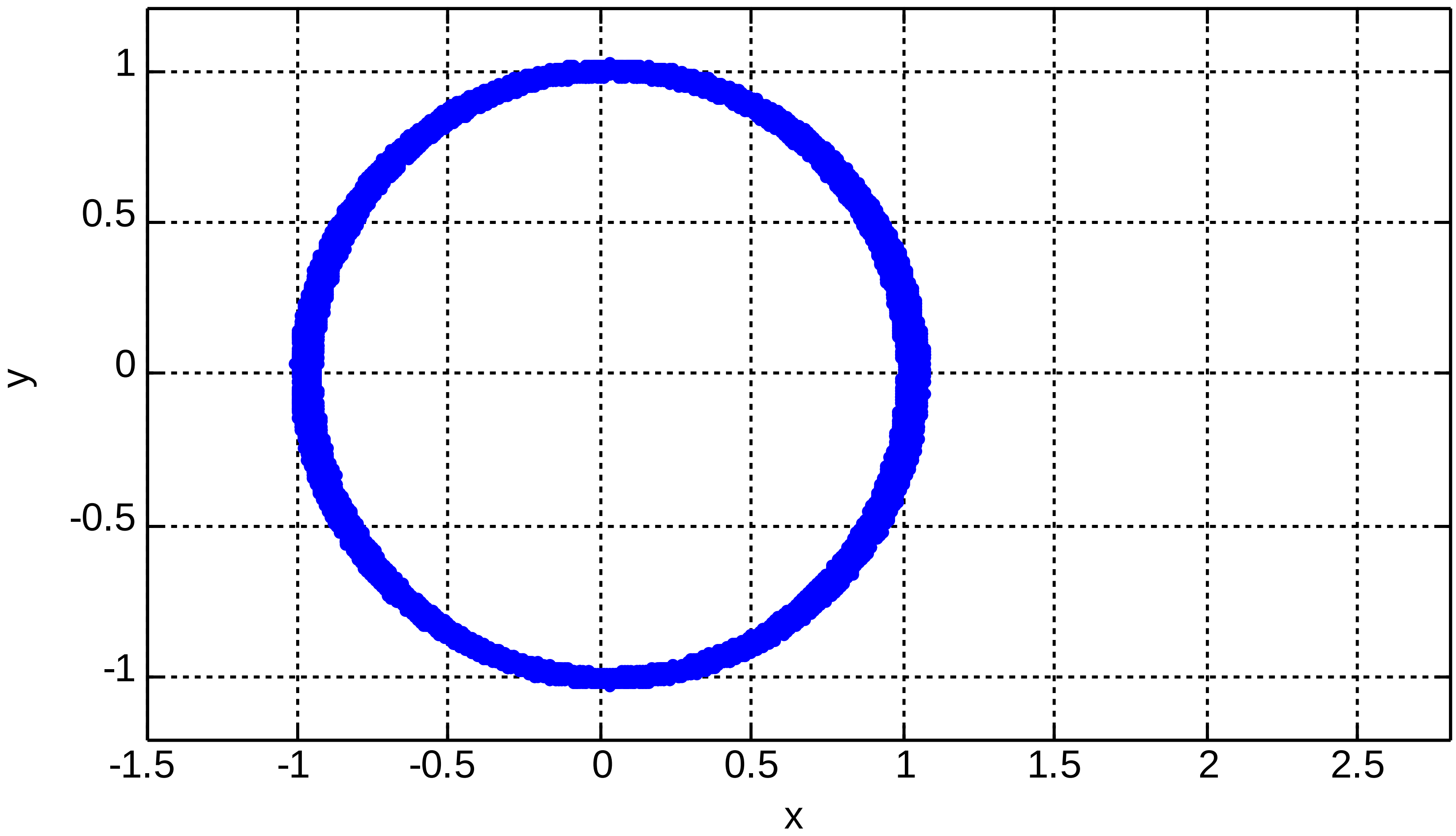
For a ray to be seen by the observer, two conditions must be satisfied: it must avoid the lens and it must reach O within the angular resolution ζ of the detector.

The appearance of a ring can be drawn once O, L, S, λ, ω and ζ are given by generating rays emitted from S at angle (θ,φ) and checking whether they obey the above conditions. Out of the SOL plane, (θ,φ) is chosen to comfortably bracket the $[\theta_+,\theta_-]$ interval.

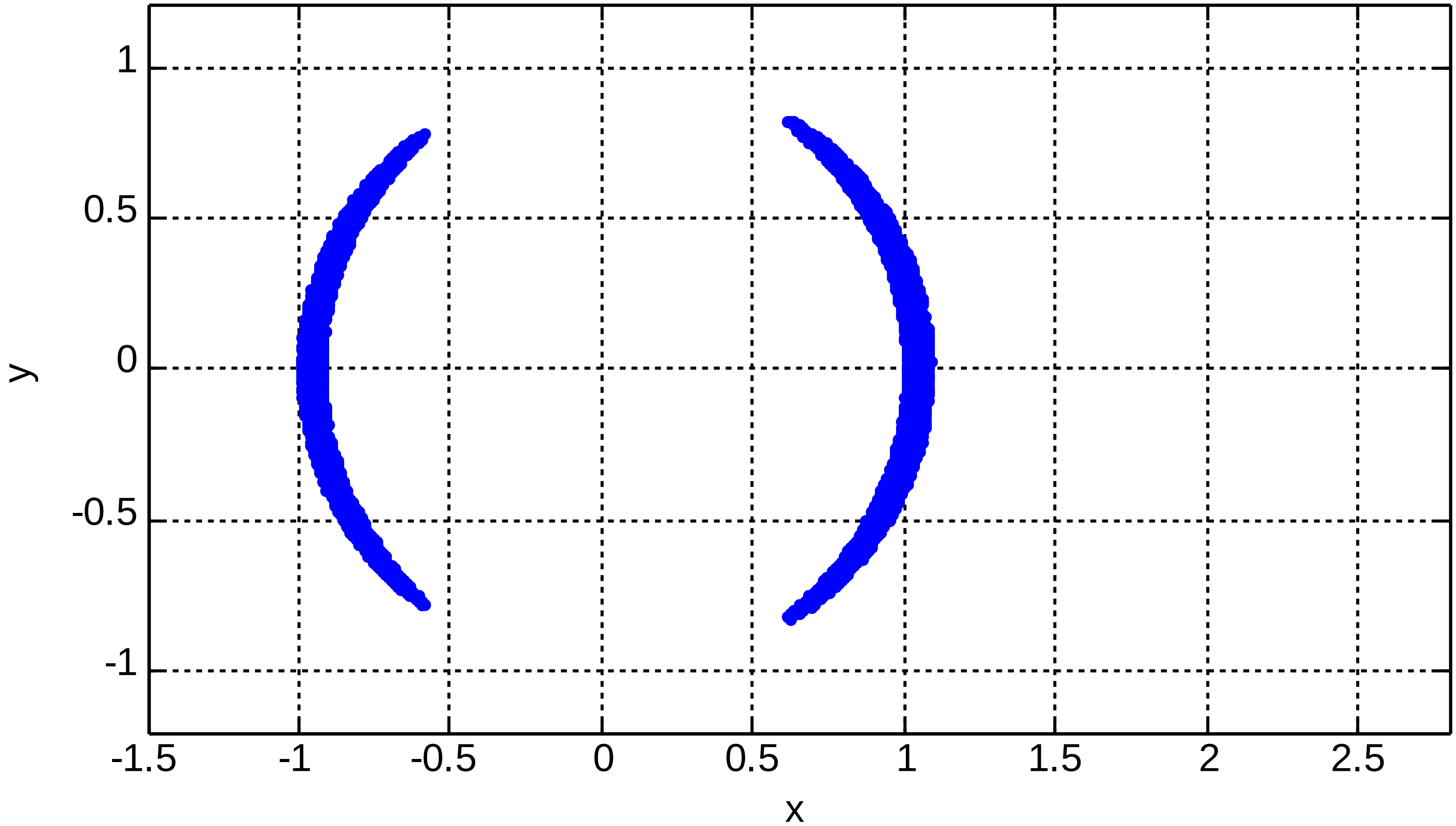
The result depends on the product $\lambda\sigma_{OS}$ and not on λ and σ_{OS} separately. Lensing a remote quasar by a galaxy ($\lambda \sim 10^{-6}$, $\sigma_{OS} \sim 10^{-4}$) or a nearby star by a foreground stellar black hole ($\lambda \sim 1$, $\sigma_{OS} \sim 10^{-10}$) gives the same ring.



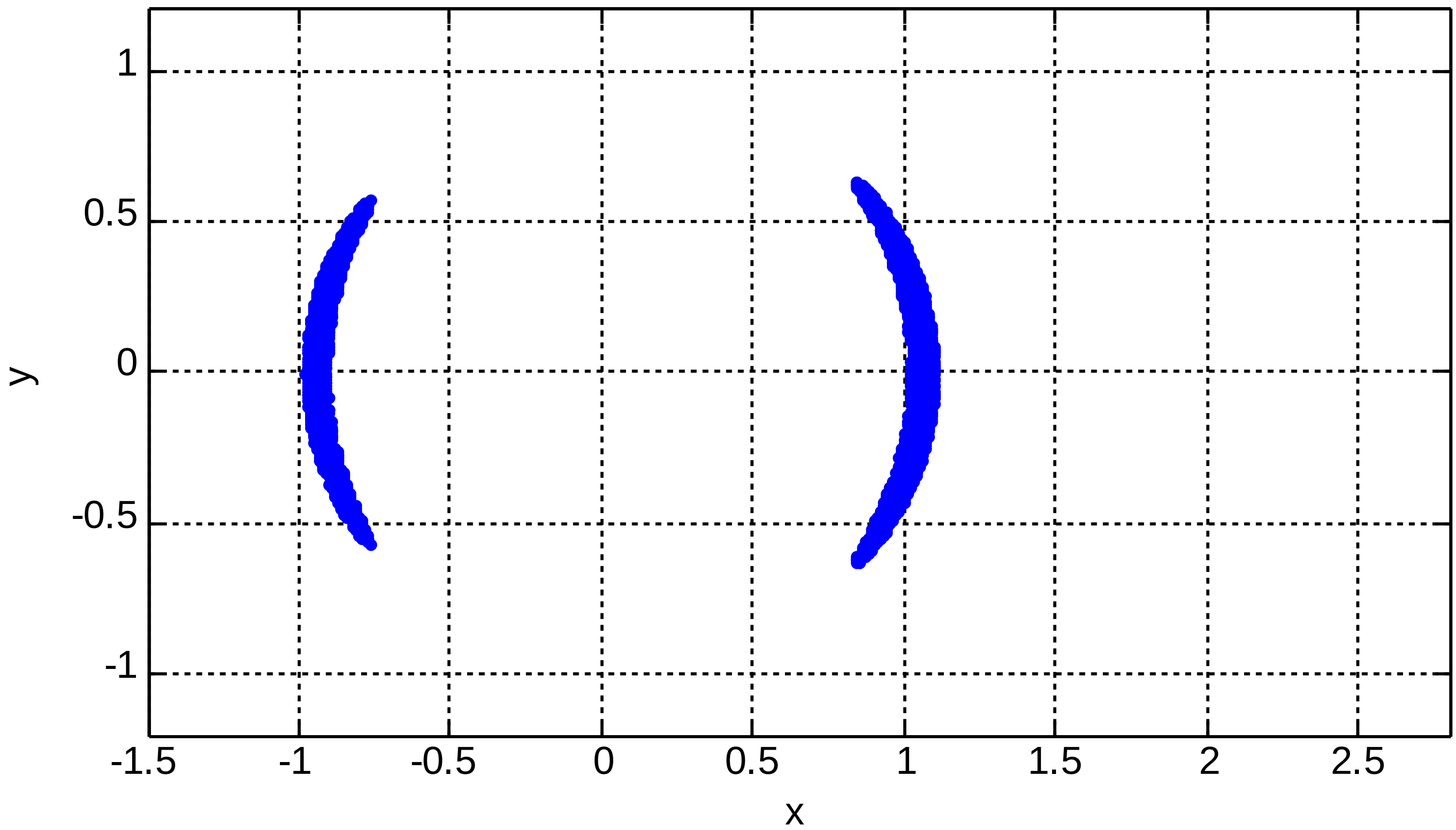
$\omega = 0$ ppm



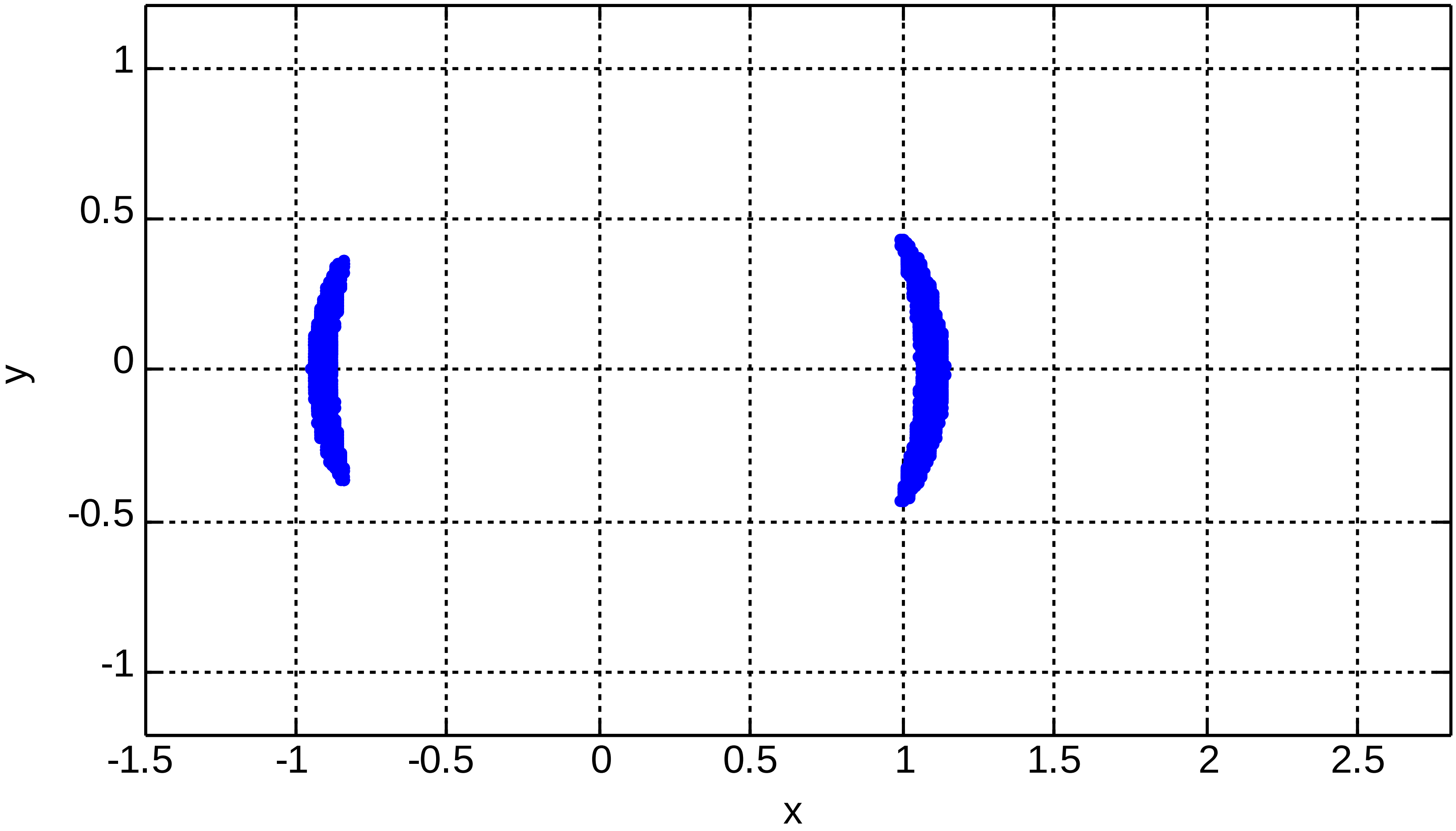
$\omega = 5$ ppm



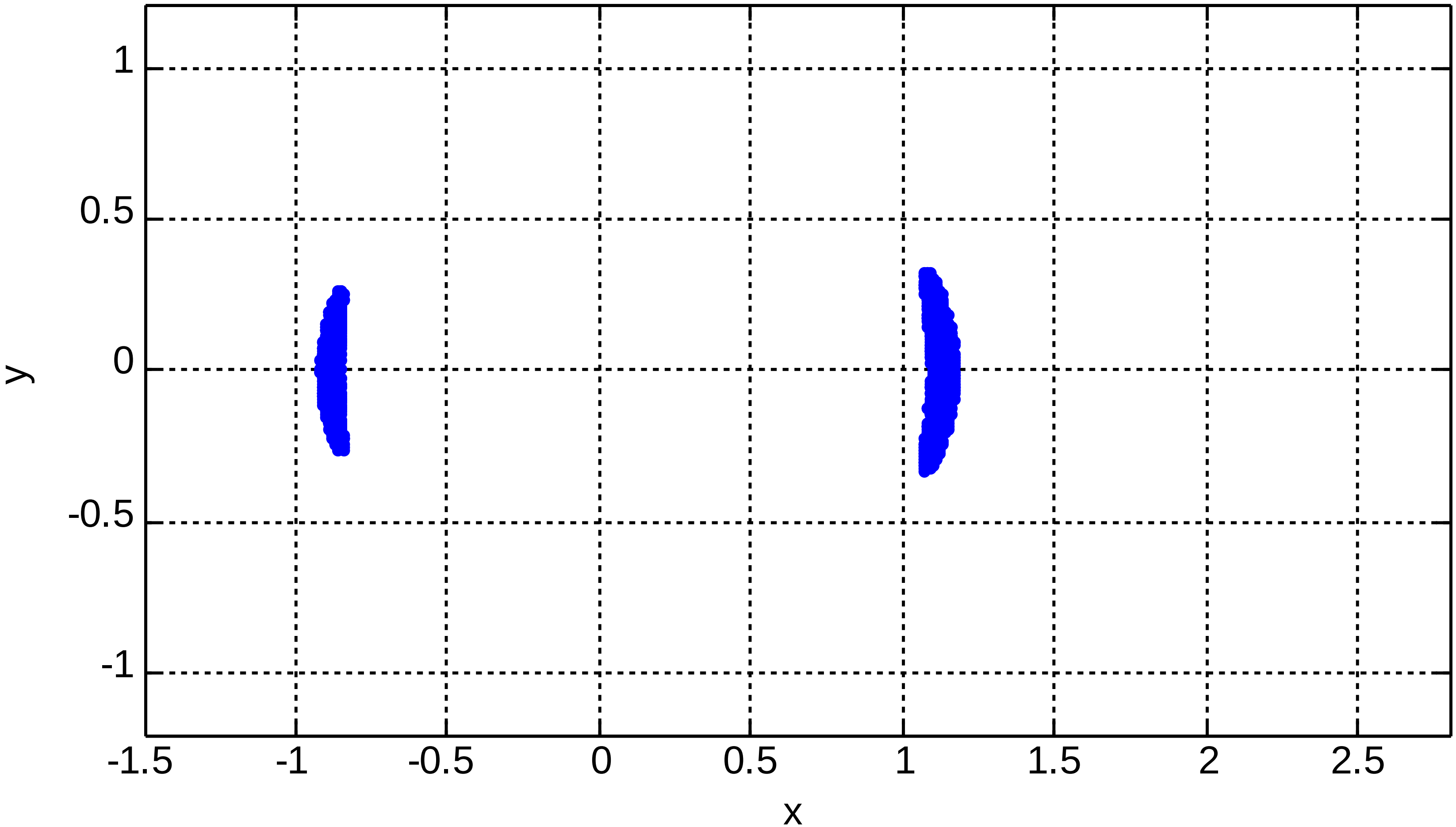
$\omega = 7.5$ ppm



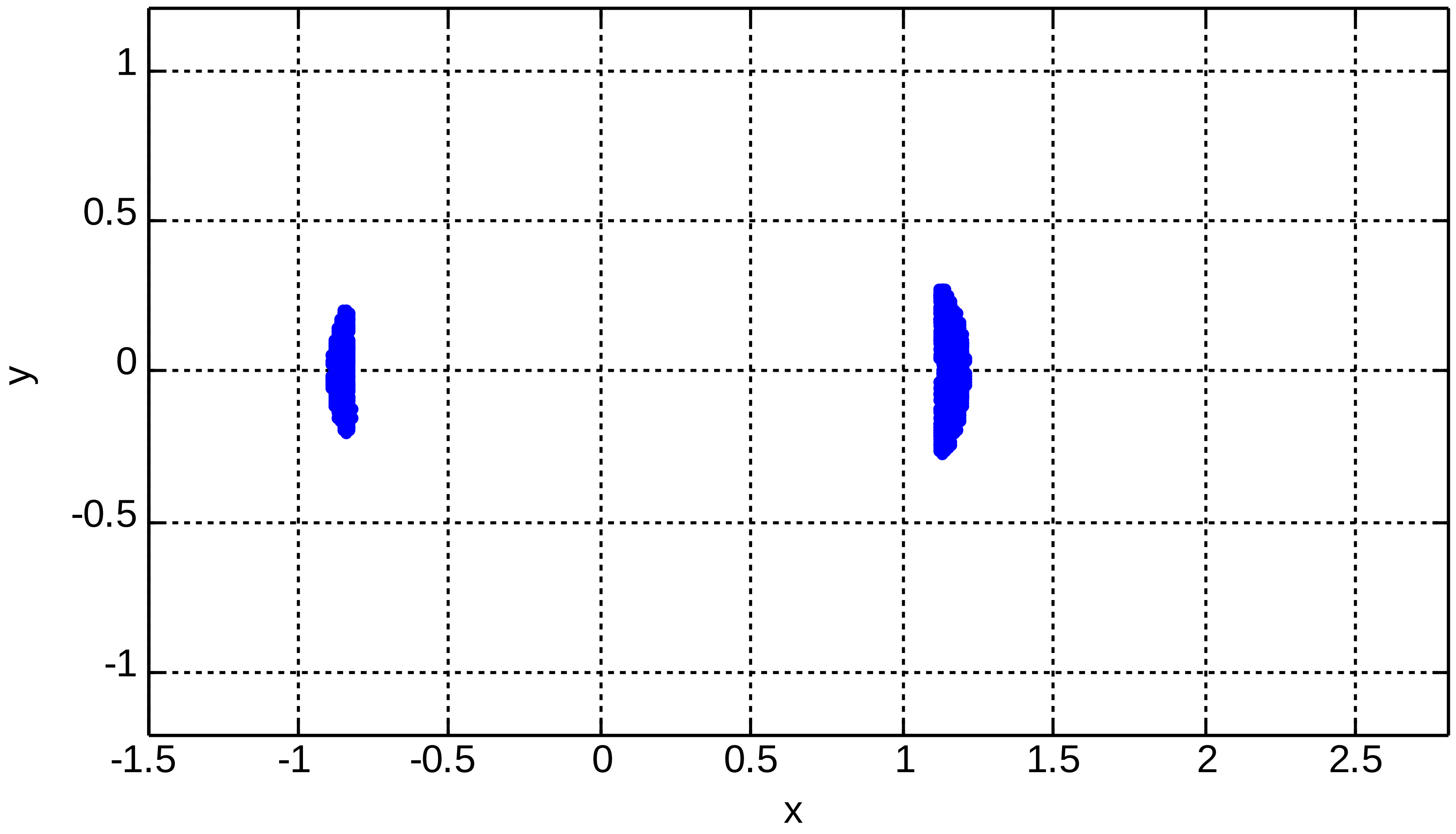
$\omega = 10$ ppm



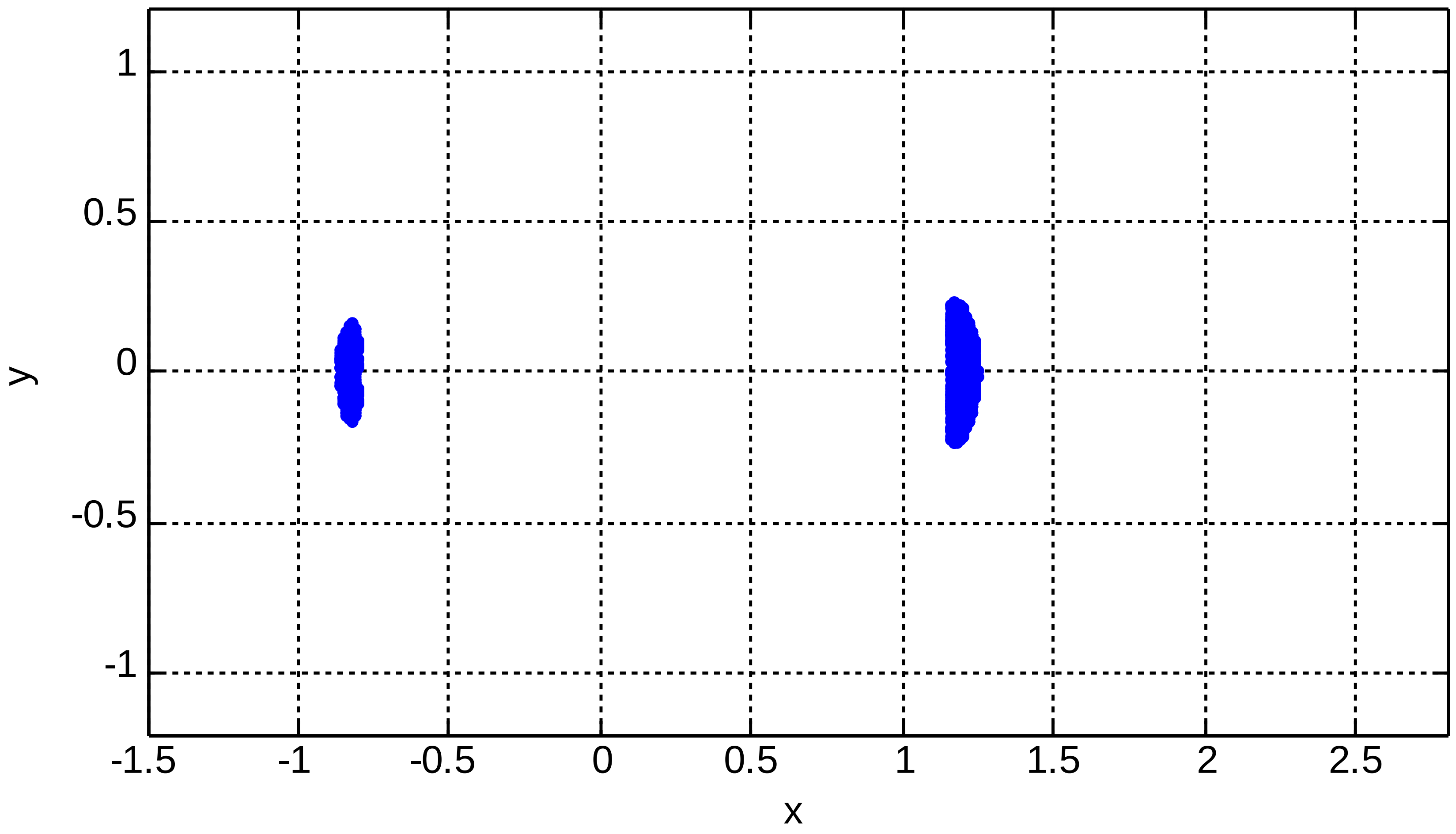
$\omega = 15$ ppm



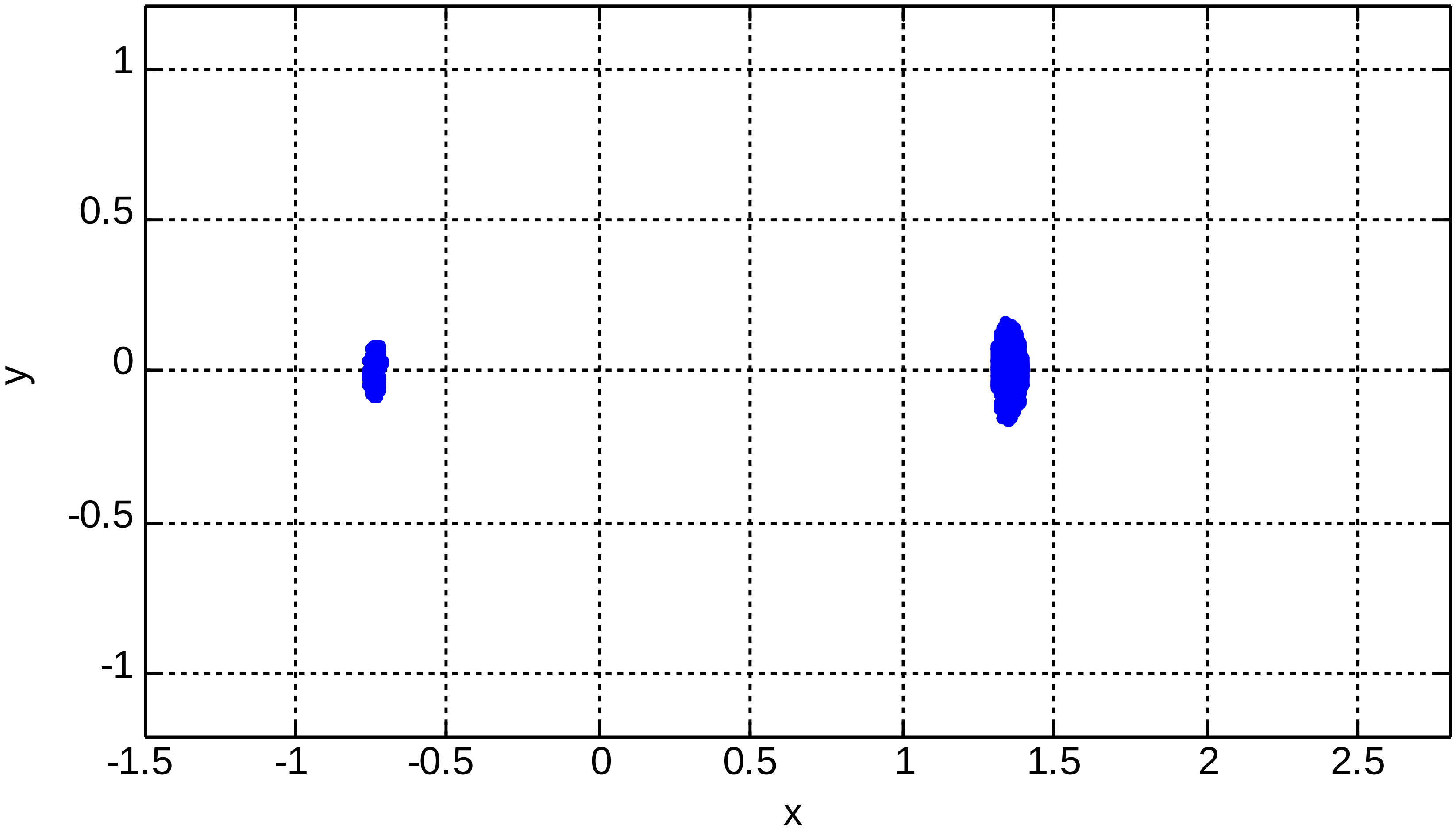
$\omega = 20$ ppm



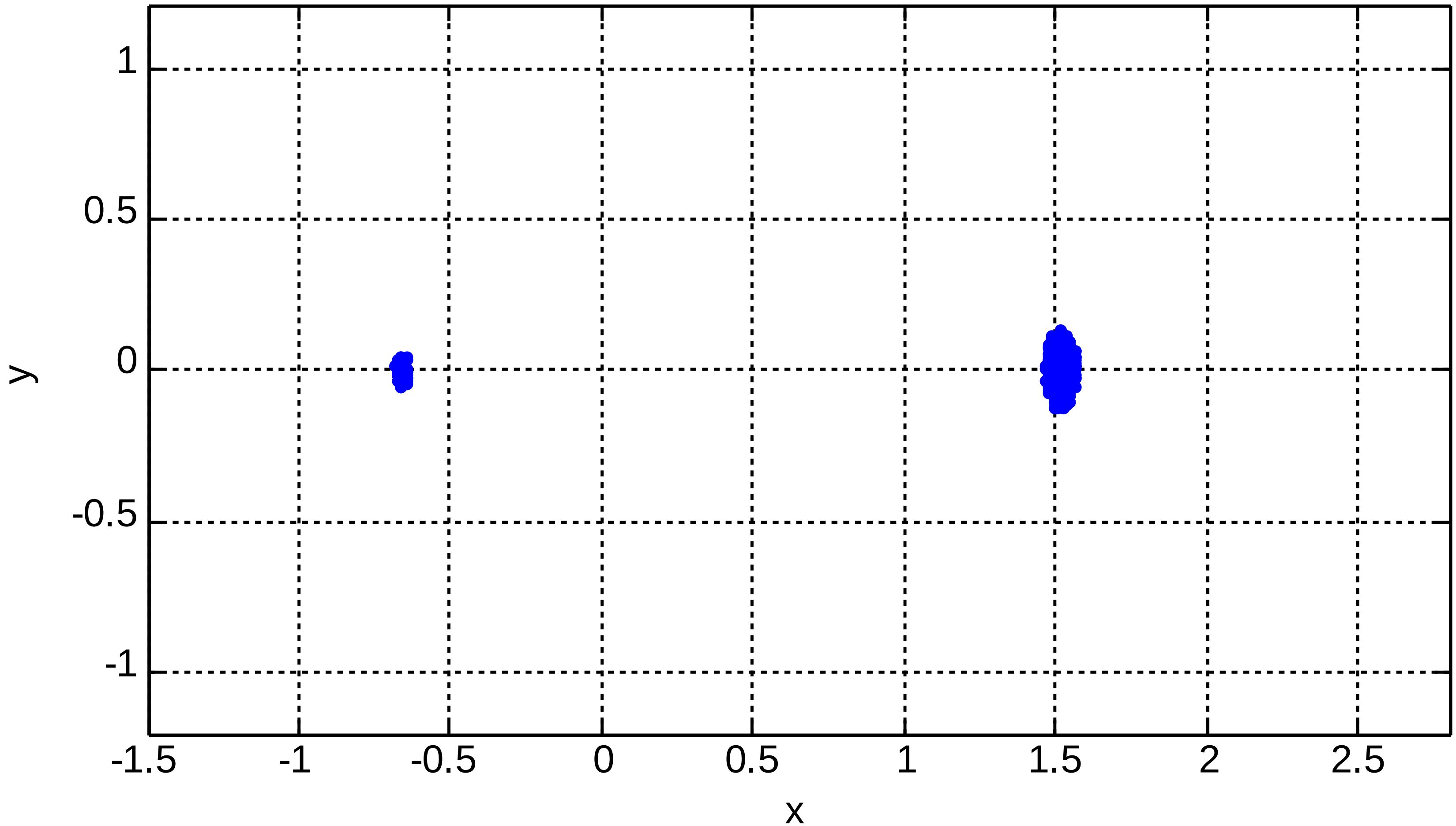
$\omega = 25$ ppm



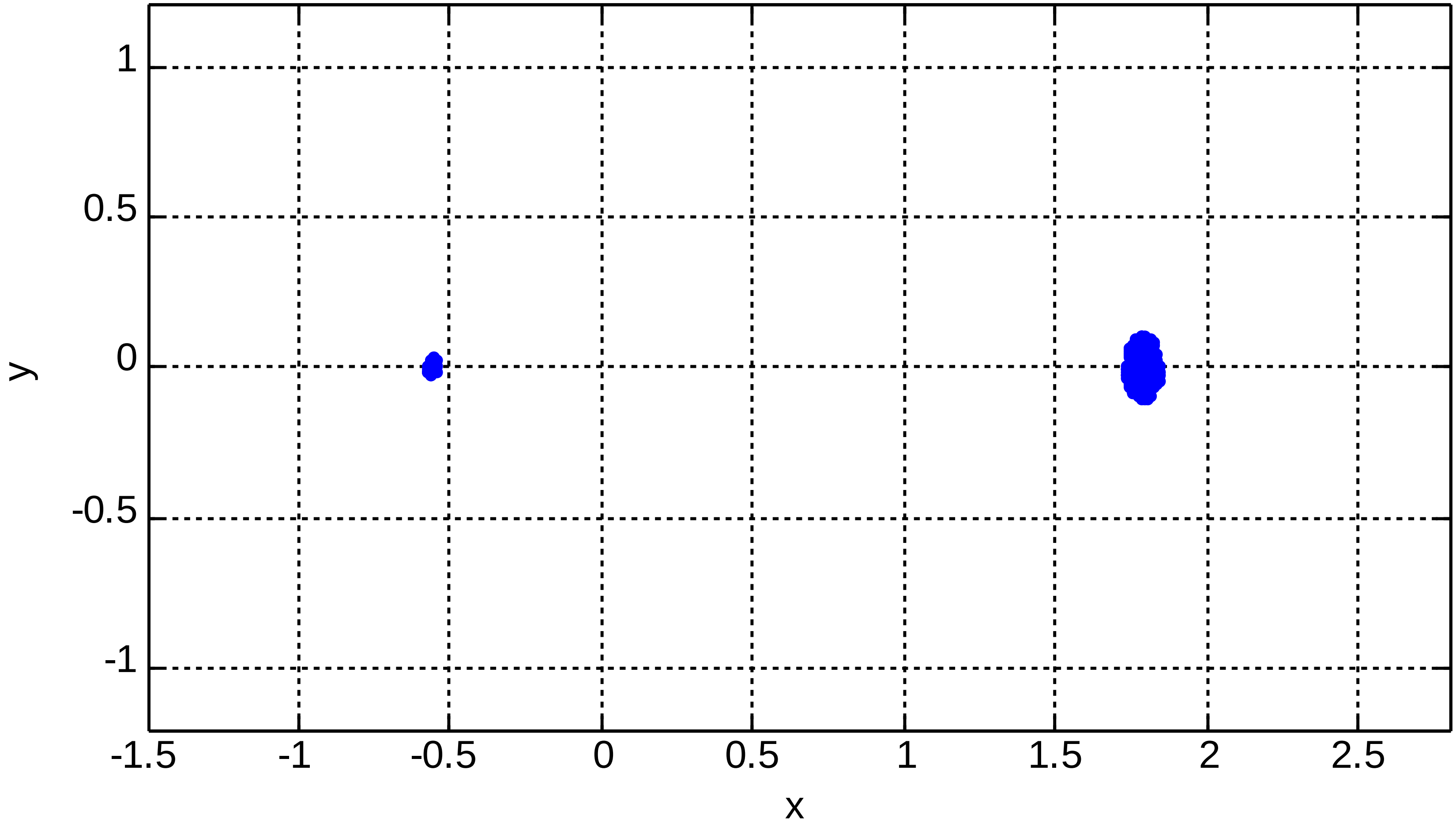
$\omega = 30$ ppm



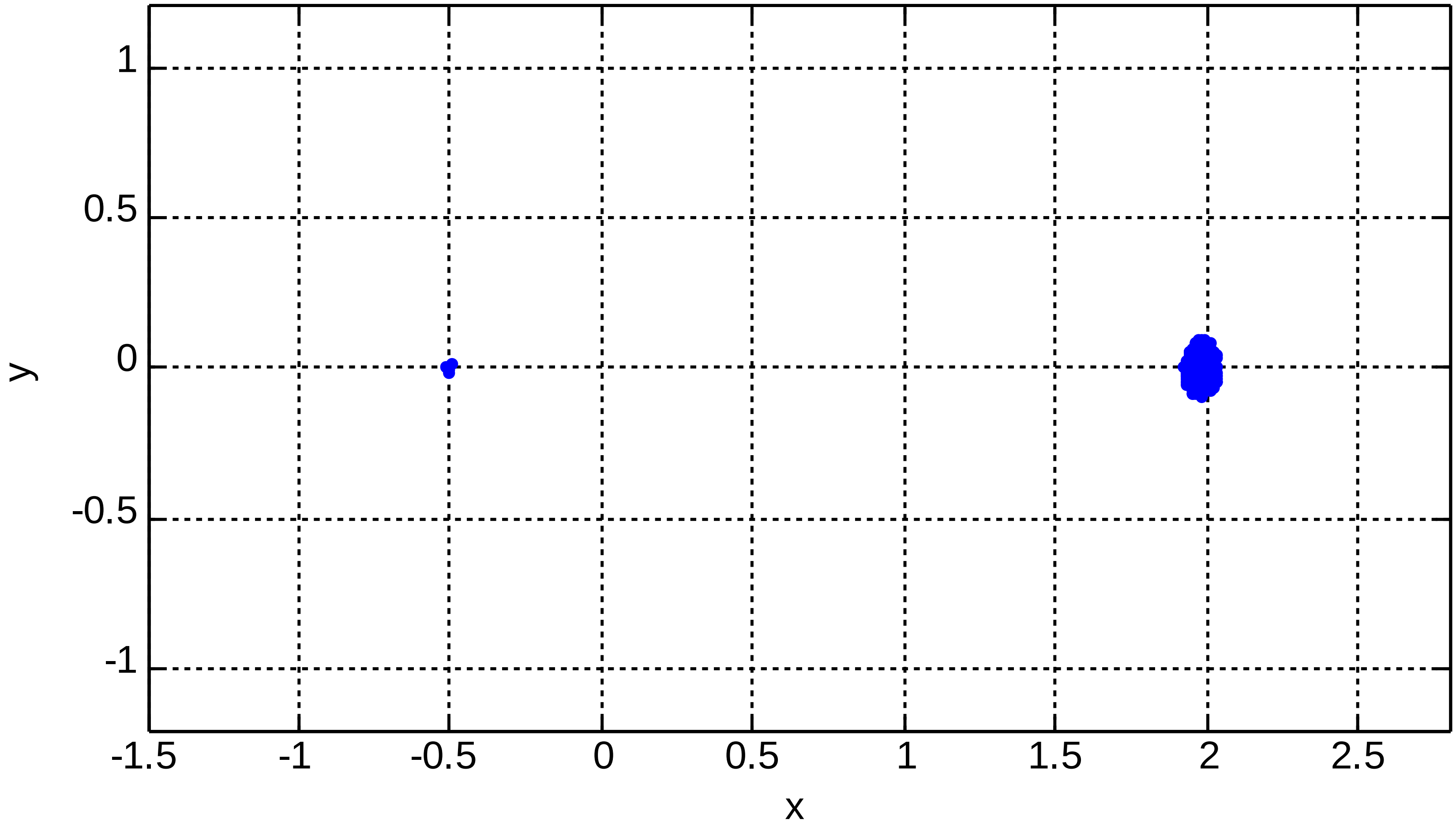
$\omega = 50$ ppm



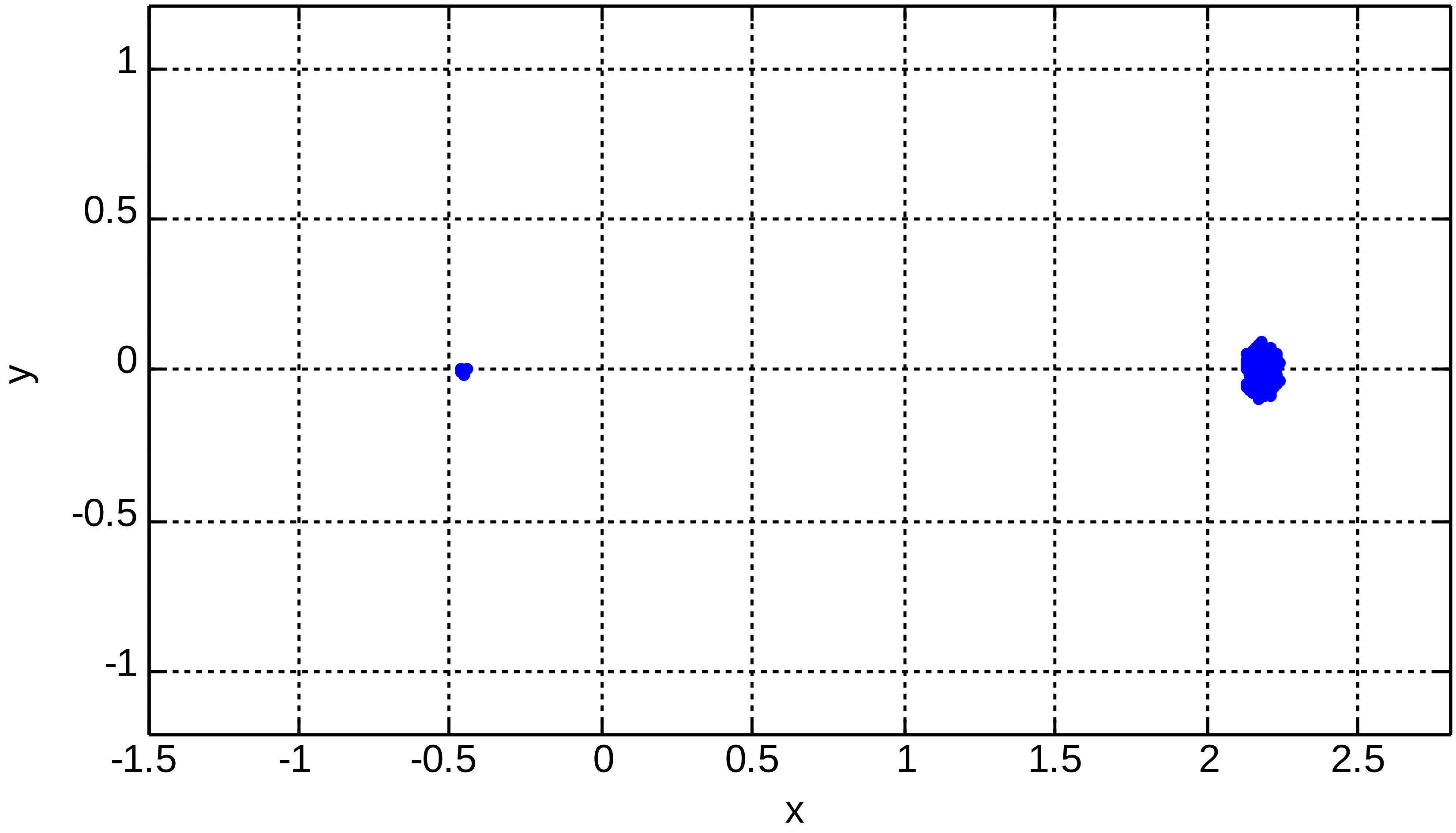
$\omega = 70$ ppm



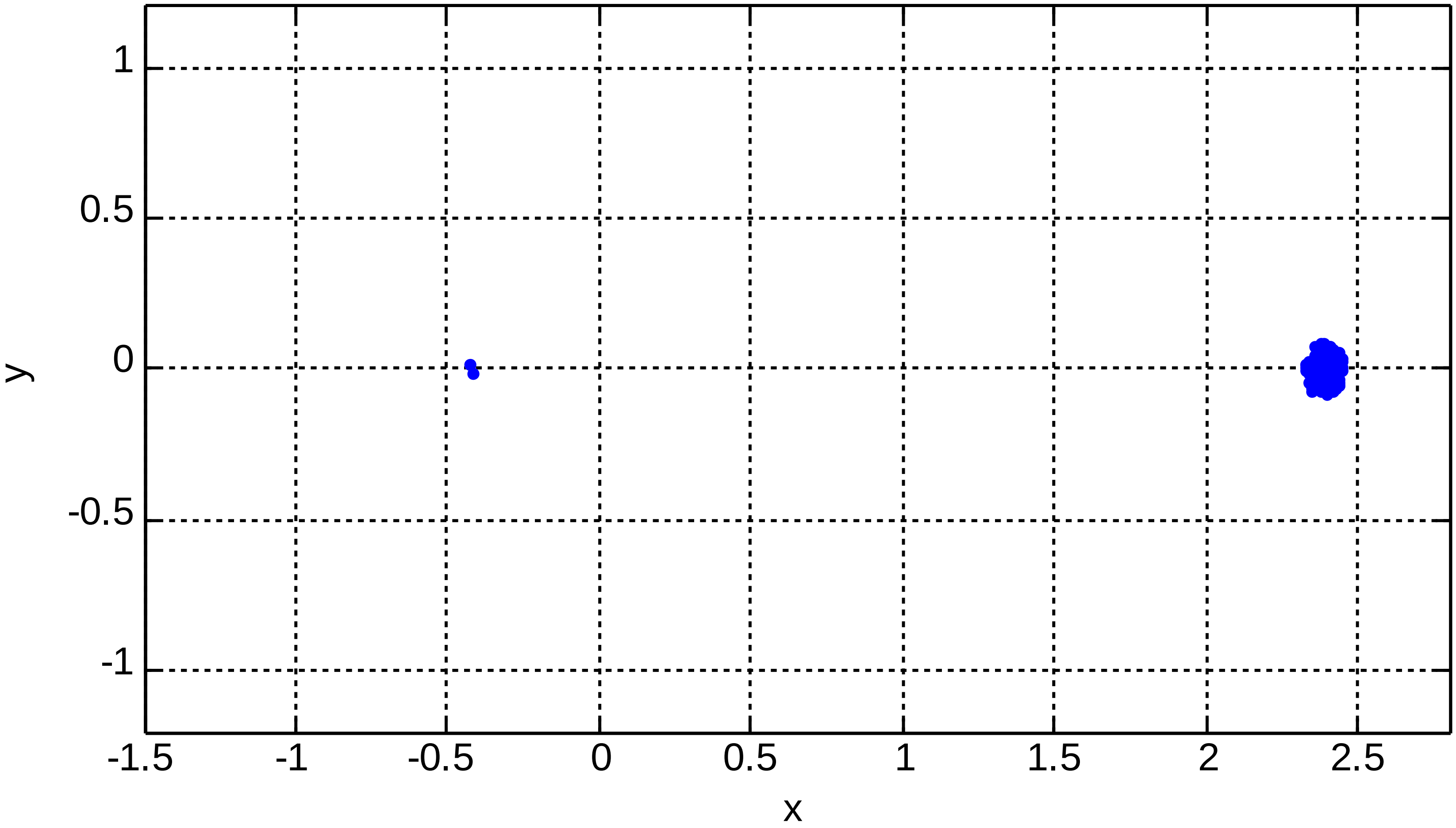
$\omega = 100$ ppm



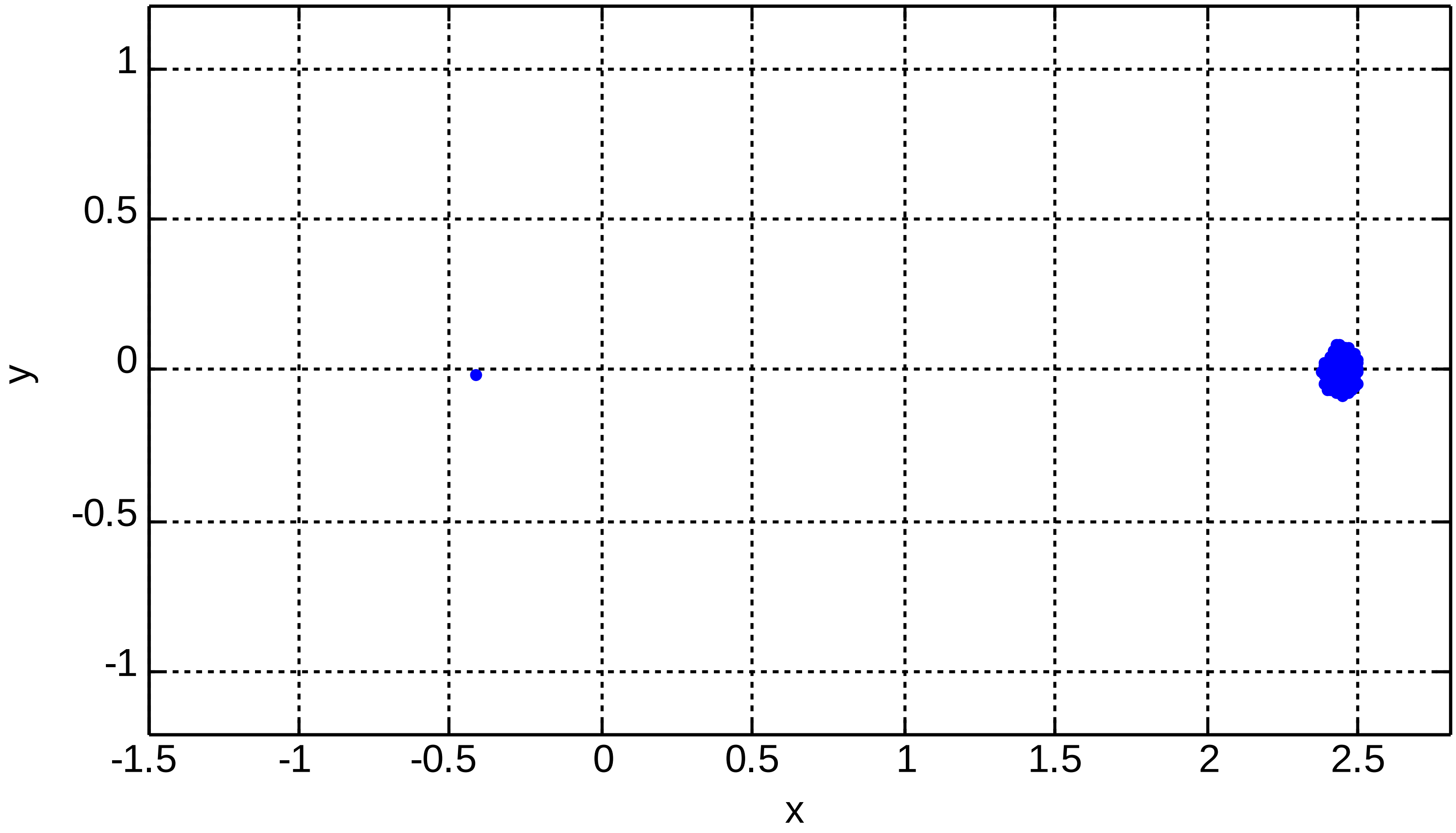
$\omega = 120$ ppm



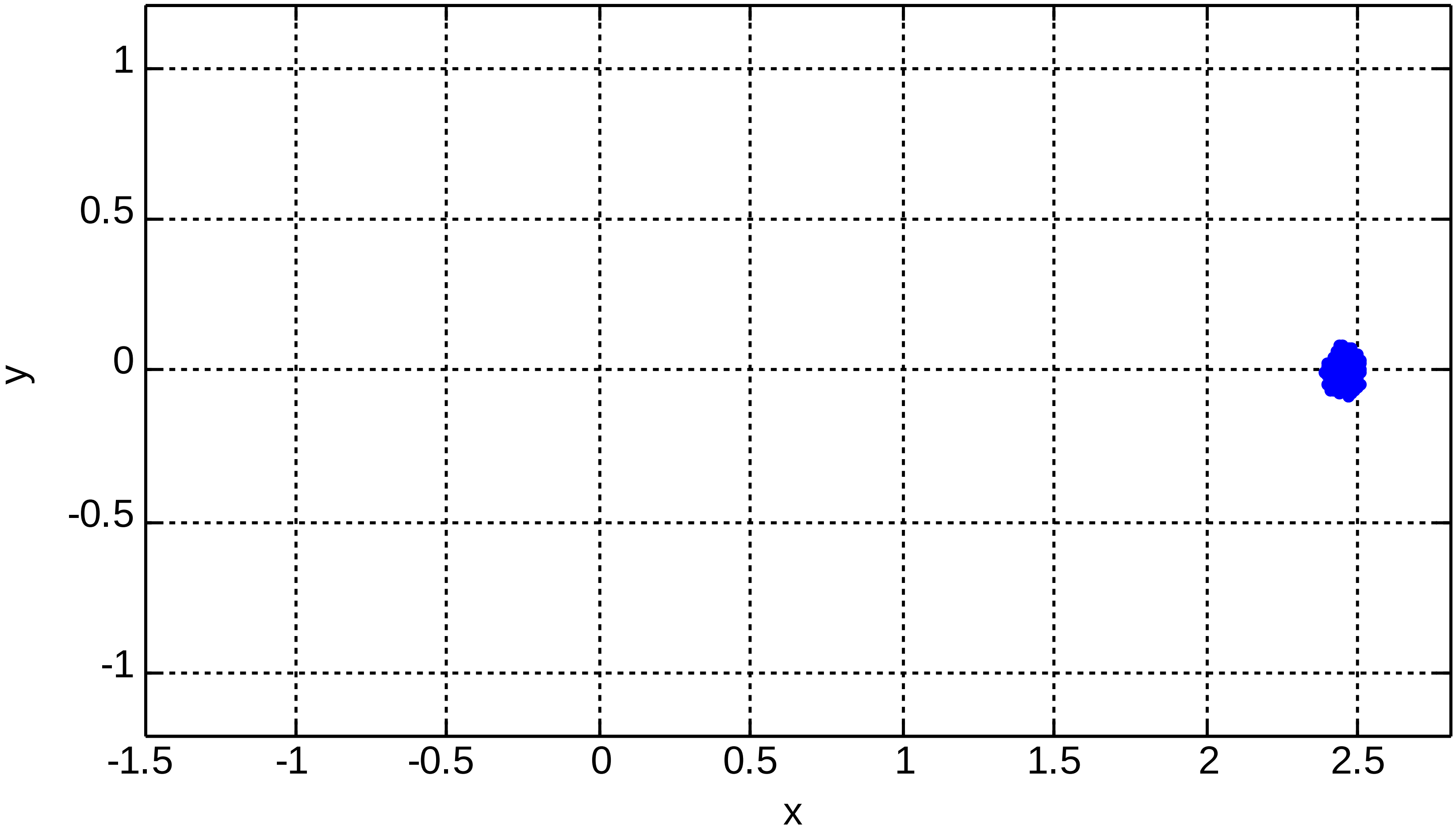
$\omega = 140$ ppm



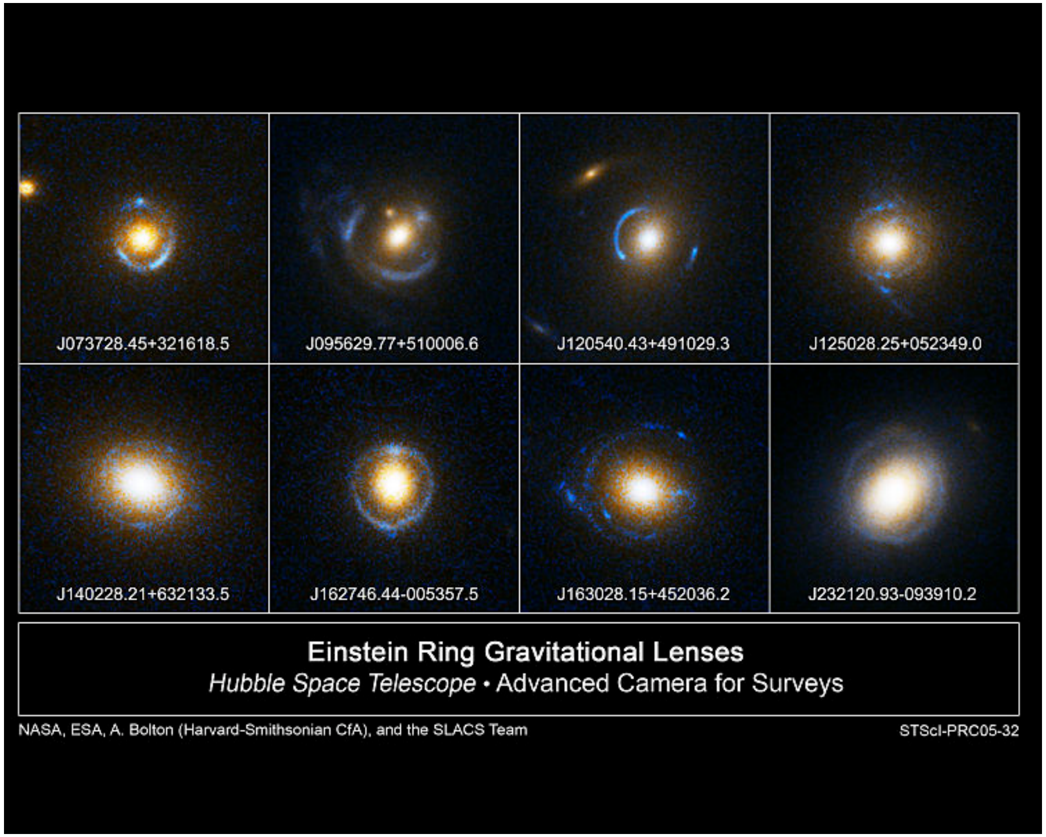
ω = 160 ppm

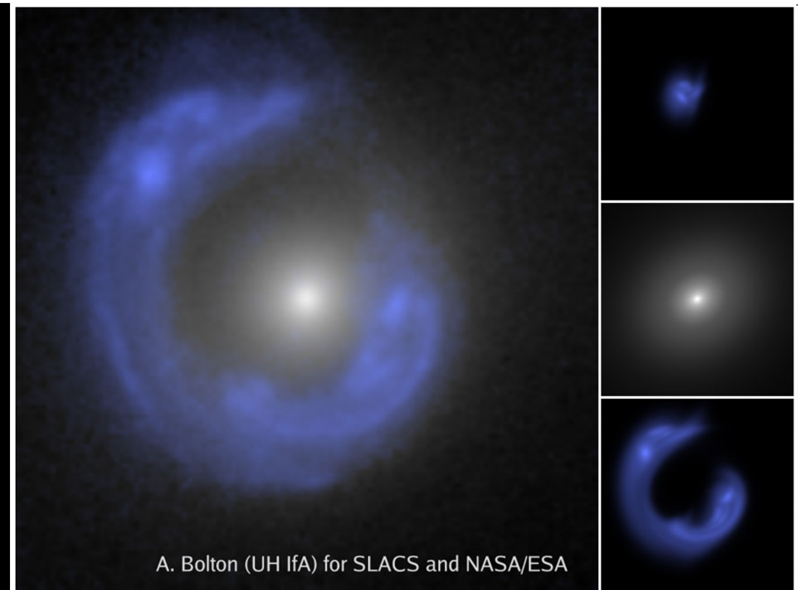


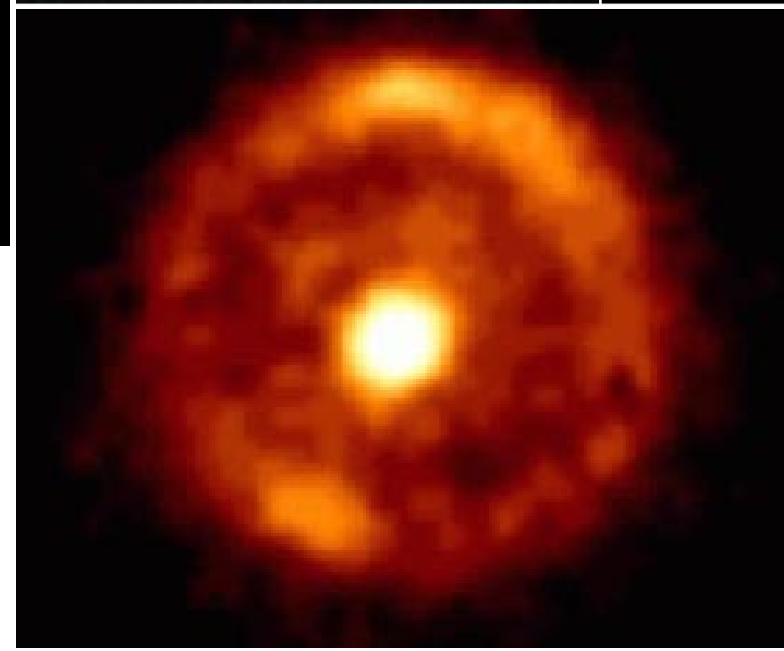
$\omega = 165$ ppm



$\omega = 166$ ppm

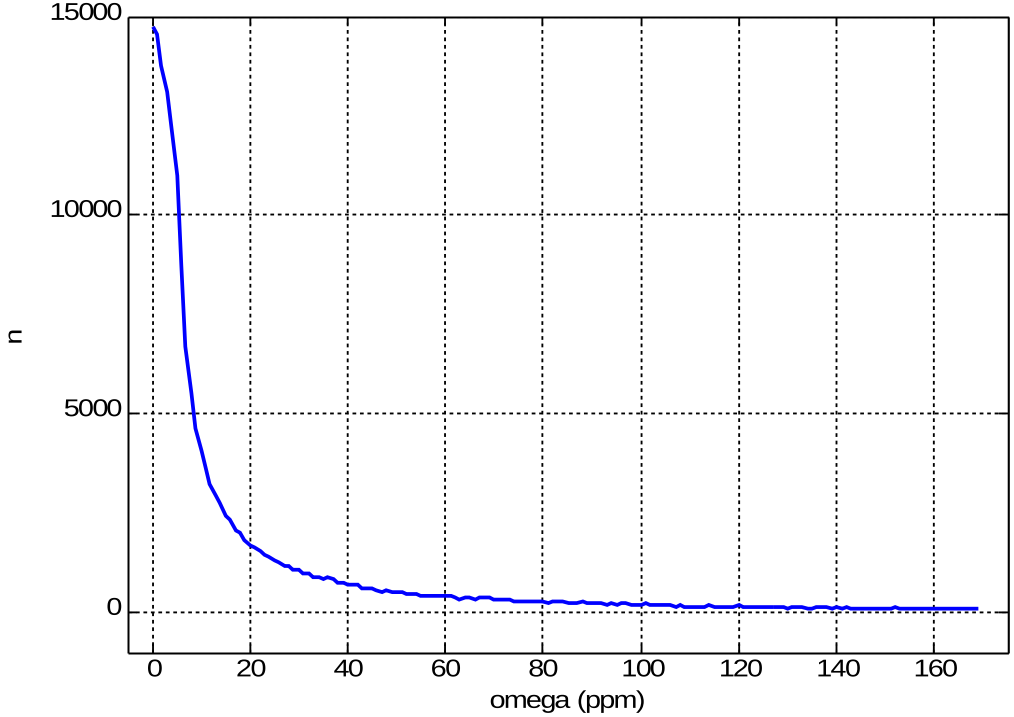




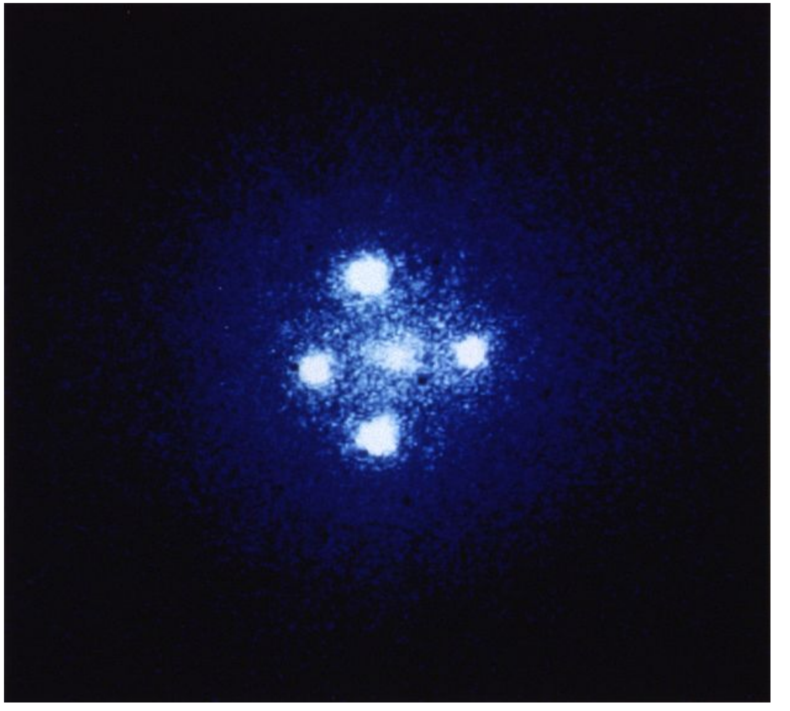


A collection of Einstein rings observed with the Hubble Space Telescope.

There is an important amplification of the light collected near perfect alignment. In the case studied here, the effect persists for values of ω of the order of 20 μrad.



In practical cases, the non-spherical form of the lens, and to a lesser extent possibly of the source, result in strongly distorted rings, which may take shapes as seen in the **Einstein Cross** or other similar images.



The simulation becomes in such cases much more complicated, each ray must be followed along its path across the complex gravitational field, but this complication is purely technical and of little interest from a physics point of view. For this reason, we restricted the present study to the case of spherical lenses and point like sources, which display the main features of gravitational lensing in a most transparent way.

Conclusions

A review of gravitational lensing has been presented including strong lensing, weak lensing and micro-lensing. The physics bases of the effect have been briefly reviewed.

Codes have been written in order to illustrate the behaviour of light in a gravitational field.

– A first code has made it possible to trace rays in the vicinity of a massive lens, with particular emphasis on the extreme bending that occurs in the vicinity of a black hole.

– A second code has been used to illustrate the formation of an Einstein ring and its disappearance as the alignment deteriorates. The light amplification that occurs in the case of perfect alignment has been demonstrated.

Thank you for your attention!